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Equivalent Circuit Parameters of S-band 1.5 Cell RF Gun Cavity

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Abstract

We determined equivalent circuit parameters of a 1.5 cell S-band RF gun cavity from the resonant characteristics of its decoupled cavities(half cell and full cell) using the code SUPERFISH. Equivalent circuit parameters of the 1.5 cell RF gun cavity resonated in the 0-mode were obtained easily from the circuit parameters of each decoupled cavities. In order to obtain equivalent circuit parameters for the π-mode cavity, we calculated the differences of the resonant frequencies and the equivalent resistances between the 0- and π-modes with slight variations of the radius and thickness of the coupling iris. From those differences, we obtained *R*/*Q* value and equivalent resistance of the π-mode, which are directly related to the equivalent circuit parameters of the coupled cavity. Using calculated *R*/*Q* value, we can express equivalent inductance, capacitance and resistances of the RF gun cavity resonated in the π-mode, which can be useful for analyzing coupled cavities in a steady state.

Key words : RF Gun, Coupled Cavity, 0-mode, π-mode, Equivalent Circuit.

Ⅰ. Introduction

It is required very high brightness electron beams for the next generation free electron lasers(FELs), future high energy linear colliders and other advanced accelerator applications. Thus, many world wide researches have undertaken developments of the RF guns which can produce electron beams with high quality[1]. While direct application of the code SUPERFISH[2] in designing RF electron gun cavity is useful for obtaining detailed information about the characteristics of the RF gun cavity[3],[4], using the code only is so cumbersome and difficult to obtain a good design specification of the cavity structure. Although cavity design necessitates full considerations of complete system consisting of an RF generator(Klystron), an electron source(cathode), a transmission line or waveguide and a coupling structure that couples the electromagnetic energy from the waveguide to the RF gun cavity, understanding of resonant characteristics of(coupled) cavity itself is essential since it can offer fundamental information in designing a coupling aperture and tuning parts as well as in a beam quality[5]. Thus, it would very helpful to have fundamental concept on resonant characteristics of coupled cavity at an initial design stage. The RF gun that we want to analyze here is the traditional BNL (Brookhaven National Laboratory) style[6]. Fig. 1 shows the cross sectional view of the 1.5 cell RF gun which is loaded with a rectangular waveguide via aperture coupling. Traditionally, most RF guns have been designed with 2D simulation tool such as SUPERFISH for predicting resonant characteristics of the axis-symmetric gun cavity it self and with 3D simulation tool such as HFSS for more detailed electromagnetic resonant characteristics of the RF gun including coupling aperture(axis-symmetric breaking structure).

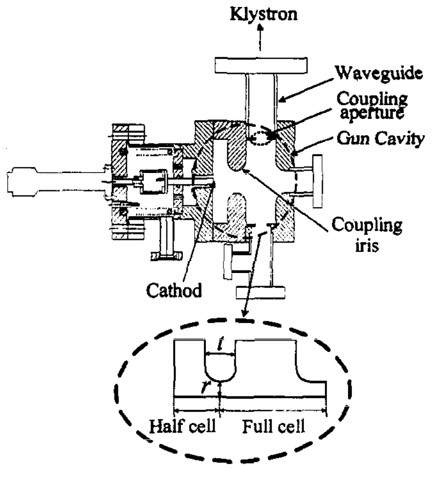

Fig. 1. Cross sectional view of the 1.5 cell RF gun loaded with a rectangular waveguide to the full cell.

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However, this design procedure includes tedious "trial and error" process. Lin et al. conducted equivalent network analysis of 1.5 cell RF gun cavity with aperture coupling[7]. Lin's complex π-mode equivalent network was highly focused on the effects of the coupling iris and coupling aperture. In this work, simple alternative equivalent circuit parameters of S-band 1.5 cell π-mode coupled RF gun cavity are obtained which can be expressed with the 0-mode parameters and the dimensions of the coupling iris, which can be useful tool for understanding the coupled cavity resonators in a steady state.

II. Resonant Characteristic Analysis of the RF Gun with the Code SUPERFISH

There exist two resonant modes in a 1.5 cell RF gun cavity, which are a "0-mode" and a "π-mode" where the phase differences are "0" and "π" radians between the half and full cells. Fig. 2 shows typical electric field pattern for the 0- and π-modes. If the RF gun cavity is resonated in the π-mode, electrons emitted from the cathode of the half cell will be accelerated by the quasi-TM_{010} fields of the half cell and then to be accelerated in the full cell because the full cell will be phased to reverse direction when electrons enter it. Therefore, the 1.5 cell RF gun cavity should be resonated to the π-mode. The preliminary SUPERFISH simulations have been conducted for the 2.856 GHz (resonant frequency of the π-mode) RF gun cavity and two decoupled cavities. Fig. 3 shows the schematic cross sectional view of the simulated cavity structures.

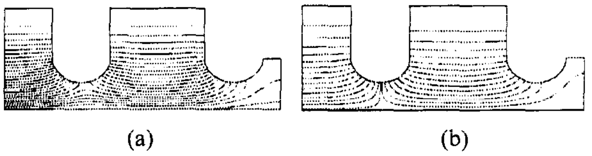
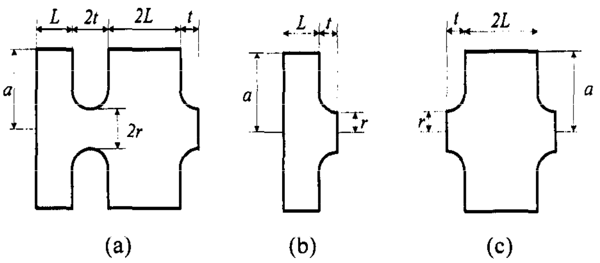

(a) (b)

Fig. 2. Electric field pattern of the RF gun cavity. (a) 0-mode, (b) π-mode



(a) (b) (c)

Fig. 3. Coupled and decoupled cavities. (a) Coupled cavity, (b) Half cell, (c) Full cell

Table 1. The resonant characteristics of the RF gun cavity and its uncoupled cavities.

	Half cell	Full cell	0-mode	π-mode
f_r [MHz]	2854.083	2854.097	2854.102	2856.004
U [μJ]	255.262	510.955	766.270	979.438
P [W]	486.394	619.639	1104.994	1643.196
Q	9411.2	14787.4	12435.7	10696.1
r_s [MΩ]	1.417	4.448	5.612	3.774

The simulation results are summarized in Table 1. In that simulation, radius and thickness of the coupling iris were r=1.0 *cm* and t=2.0 *cm*, respectively. Lengths of half and full cells were $L+t$=2.625 *cm* and $2(L+t)$=5.350 *cm*, respectively. The radius of cavity was a=4.180 *cm*. In Table 1, f_r, U, P, Q, and r_s are resonant frequency, stored energy, dissipated power, quality factor and shunt impedance, respectively.

The resonant frequency of the 0-mode was identical to two decoupled cavities. The stored energy, dissipated power, and shunt impedance of the 0-mode are simple sums of those of two decoupled cavities. The quality factor of the 0-mode coupled cavity is the arithmetic mean of those of the decoupled cavities. The errors are less than 1 % for all resonant characteristics between 0-mode coupled cavity and decoupled cavities listed in Table 1. As shown in Fig. 2(a), in the 0-mode case, we can decouple the coupled cavity to two independent decoupled cavities with half of coupling iris without disturbing modal field patterns since electric field lines are smoothly connected between two cells except near the coupling iris. Thus, the resonant characteristics of two decoupled cavities are directly related to those of the coupled cavity. However, in the π-mode case, we cannot decouple the cavity having identical modal field patterns. Thus, the resonant characteristics of the π-mode cavity is complicated and difficult to describe with those of decoupled cavities. The relations between the 0- and π-modes will be considered in later section with equivalent circuit parameters.

The simulation is also conducted by slightly changing the radius and thickness of the coupling iris shown in Fig. 1. As the radius and thickness of the iris increase, the resonant frequencies also increase. But the difference of the resonant frequencies between the two resonant modes increases as the radius of the iris increases and the thickness of the iris decreases.

Fig. 4 shows the changes of the two resonant

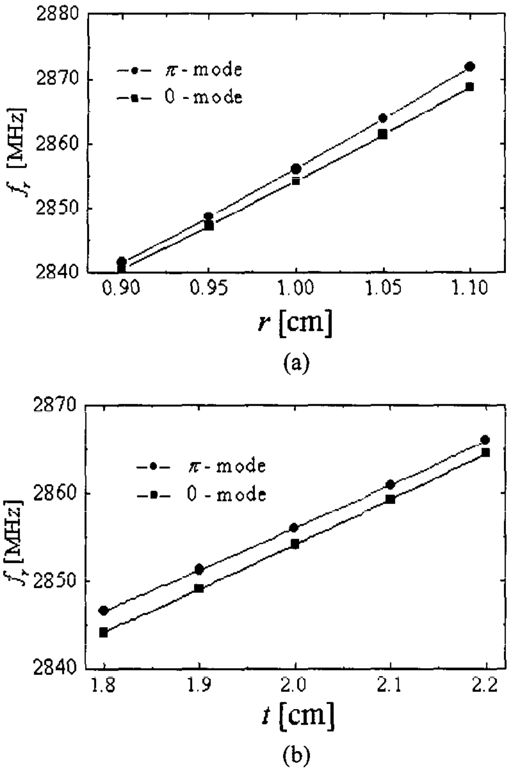


Fig. 4. The resonant frequencies (a) with different radius of the iris, (b) with different thickness of the iris.

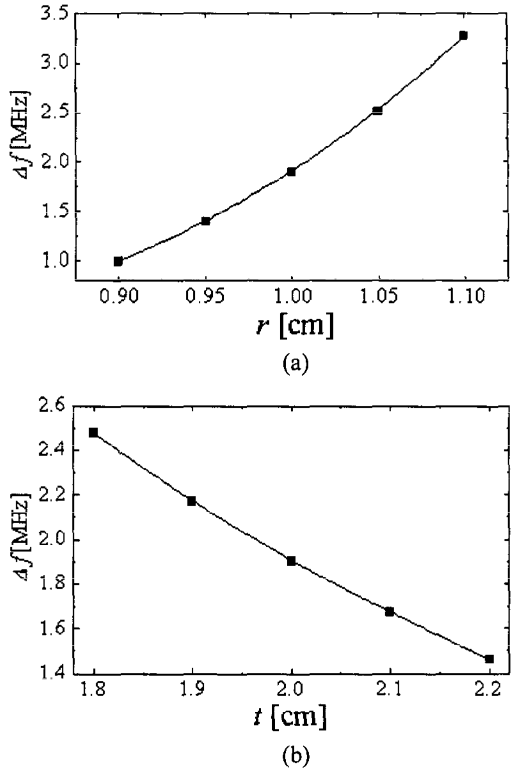


Fig. 5. The frequency difference between two resonant modes (a) with different radius of the iris, (b) With different thickness of the iris.

frequencies, i.e., 0- and π-modes with respect to the radius and thickness of the coupling iris. Fig. 5 shows the difference of the resonant frequency between the 0- and π-modes. In the case of the 0-mode coupled cavity and two decoupled cavities, the resonant characteristics between them we mentioned previously were still valid within 1% error even slightly altering the radius and thickness of the coupling iris. From Fig. 5, we obtained the approximated expression of the frequency difference relation which depends on the thickness and the radius of the iris as

$$\Delta f = f_\pi - f_0 \cong Kr^x - t^{-y} \tag{1}$$

where Δf is the frequency difference between the π-mode and the 0-mode, f_π and f_0 are the resonant frequencies of the π-mode and the 0-mode, respectively. The frequency difference depends on the dimensions of the coupling iris. x, y and K are constants, and r and t are the radius and thickness of the iris, respectively. As we previously mentioned, the RF gun should be operated in the π-mode. However, if the frequency of the source(Klystron) is unstable, the operation can be partly under the 0-mode frequency, which causes degradations of the beam quality. Thus, larger value of the frequency difference is desirable for the stable π-mode operation. Meanwhile, much larger value of the difference also causes the degradations of beam quality (larger value of emmitance), we need to find a optimal point, which is beyond the scope of this paper. We obtained the constants x, y, and K as x=5.945, y=2.631, and K=11.781.

III. Equivalent Circuit Parameters of the 1.5 Cell Coupled RF Gun Cavity

3-1 0-Mode

A parallel resonant circuit is the simplest model for describing a single mode of an accelerating cavity[8]. Fig. 6 shows a parallel resonant circuit. In accelerator physics, the shunt impedance(in Table 1) is defined as

$$r_s = \frac{V^2}{P} \tag{2}$$

where $V = \int_l E(0, z)\,dz$ is the axial voltage gain and is

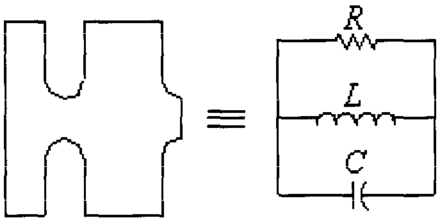


Fig. 6. Parallel resonant circuit.

the dissipated power in the cavity wall. The shunt impedance is related to the circuit shunt impedance(or simply, resistance) R as $r_s=2R$. Thus,

$$R=\frac{1}{2}r_s=\frac{V^2}{2P} \tag{3}$$

From the expressions of the quality factor for the parallel resonant circuit model[9], *i.e.*,

$$Q=\omega RC=\frac{R}{\omega L} \tag{4}$$

the equivalent inductance and the equivalent capacitance can be expressed as follows, respectively.

$$L=\frac{R}{\omega Q} \tag{5}$$

$$C=\frac{Q}{\omega R} \tag{6}$$

The resonant characteristics in Table 1 between 0-mode coupled cavity and its decoupled ones are in quite close relationships. Thus, the equivalent circuit parameters of the half cell and of full cell provide simple circuit parameters of the 1.5 cell RF gun cavity as follows.

$$L_0=L_h+L_f \tag{7}$$

$$C_0=\frac{C_hC_f}{C_h+C_f} \tag{8}$$

$$R_0=\frac{9R_fR_h}{R_f+4R_h} \tag{9}$$

Subscripts *h*, *f* and *0* denote the half cell, the full cell and the coupled cavity resonated in the 0-mode, respectively. While equivalent capacitance and inductance were expressed as in a general circuit theory, the equivalent resistance was not simply the sum of equivalent resistances of the half and full cells because the equivalent resistance is derived from the voltage-power relation as follows.

$$R_0=\frac{V_0^2}{2P_0}=\frac{(V_h+V_f)^2}{2(P_h+P_f)}$$

$$\neq\frac{V_h^2}{2P_h}+\frac{V_f^2}{2P_f}=R_h+R_f \tag{10}$$

The errors of eq. (7), (8), and (9) are less than 1 % in all cases for changing the radius and the thickness of the iris, *i.e.*, from 0.9 to 1.1 cm for the radius and from 1.8 to 2.2 cm for the thickness.

3-2 π-Mode

Unlike the 0-mode case, for the π-mode, each electric field line from the half cell does not connect to a field line from the full cell, and field lines are terminated on the coupling iris as shown in Fig. 2 (b). Thus the π-mode cannot be decoupled to two uncoupled cavities without disturbing the modal field pattern. From eq. (5) and (6), we can obtain the relation as

$$\left(\frac{R_\pi}{Q_\pi}\right)^2=\frac{L_\pi}{C_\pi} \tag{11}$$

Here, the subscript π in eq. (11) stands for the π-mode.

Since

$$\omega_\pi^2=\frac{1}{L_\pi C_\pi} \tag{12}$$

From eq. (11) and (12),

$$\left(\frac{R_\pi}{Q_\pi}\right)^2\frac{1}{\omega_\pi^2}=L_\pi^2 \tag{13}$$

Thus, we can obtain the equivalent inductance of the π-mode as

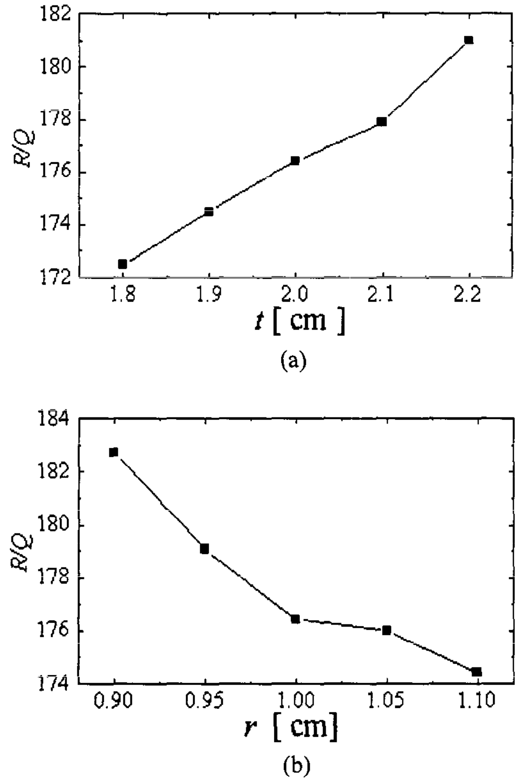
$$L_\pi=\left(\frac{R_\pi}{Q_\pi}\right)\frac{1}{\omega_\pi}=\left(\frac{R_\pi}{Q_\pi}\right)\frac{1}{\omega_0+\Delta\omega} \tag{14}$$

where ω_0 is the resonant angular frequency of the 0-mode and $\Delta\omega$ is the resonant frequency difference between π-mode and 0-modes.

Similarly, the equivalent capacitance of the π-mode is

$$C_\pi=\left(\frac{R_\pi}{Q_\pi}\right)\frac{1}{\omega_\pi}=\left(\frac{Q_\pi}{R_\pi}\right)\frac{1}{\omega_0+\Delta\omega} \tag{15}$$

Thus, in order to obtain equivalent inductance and capacitance, R/Q values should be obtained in advance. Since R/Q value is easy to measure and is independent of material loss[10]~[12], it is very useful to obtain equivalent circuit parameters of the cavity. Fig. 7 shows the R/Q value of the π-mode obtained directly from



(a)

(b)

Fig. 7. *R/Q* value of the π-mode (a) *R/Q* with different thickness of the iris, (b) *R/Q* with different radius of the iris.

the SUPERFISH simulation results. From Fig. 7, we can obtain the *R/Q* value which depends on the thickness and the radius of the iris as

$$\frac{Q_\pi}{R_\pi}=Ar^{-B}t^{C} \tag{16}$$

where A, B and C are constants and their values are shown in Table 2. Therefore, equivalent inductance and capacitance of the π-mode coupled cavity can be expressed as follows from eq. (14), (15), and (16).

$$L_\pi=\frac{Ar^{-B}t^{C}}{\frac{1}{\sqrt{L_0C_0}}+2\pi Kr^{x}t^{-y}} \tag{17}$$

$$C_\pi=\frac{(Ar^{-B}t^{C})^{-1}}{\frac{1}{\sqrt{L_0C_0}}+2\pi Kr^{x}t^{-y}} \tag{18}$$

Table 2. Constants in eq. (16).

	$1.8\leq t\leq 2.0$		$2.0\leq t\leq 2.2$	
$0.9\leq r\leq 1.0$	A	151.902	A	136.163
	B	0.370	B	0.370
	C	0.216	C	0.374
$1.0\leq r\leq 1.1$	A	151.902	A	136.163
	B	0.192	B	0.192
	C	0.216	C	0.374

Finally, let us move to the equivalent resistance of the π-mode coupled cavity. Fig. 8 and Fig. 9 show the equivalent resistance of the π-mode and the difference of the equivalent resistances between the 0-mode and the π-mode, respectively from also direct SUPERFISH simulation results. Together with Fig. 8 and 9, we obtain the expression of the differences of the equivalent resistances between the 0- and the π-modes and the equivalent resistance of the π-mode coupled cavity as shown in Table 3 and 4, which were obtained from curve fittings. The errors of the eq. (17), eq. (18), and expressions in Table 4 were less than 1 % in all cases for changing the dimensions of the coupling iris in Table 4. From the expressions of eq. (17), eq. (18), and Table 4, we can construct an equivalent circuit model of the 1.5 cell RF gun resonated in the π-mode. The equivalent circuit is composed of the functions depend on the thickness and the radius of the iris and

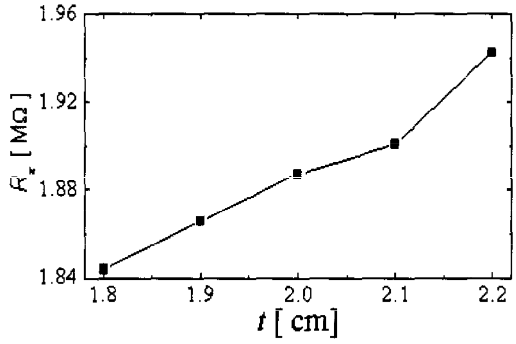


Fig. 8. Equivalent resistance of the π-mode depends on the thickness of the iris.

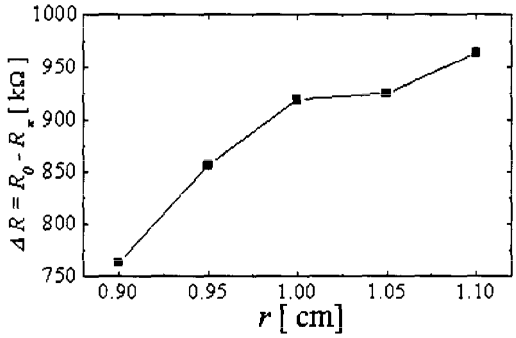


Fig. 9. The difference of the resistances between the 0-mode and the π-mode depends on the radius of the iris.

Table 3. The difference of the resistance between the 0-mode and the π-mode.

	$1.8 \leq t \leq 2.0$	$2.0 \leq t \leq 2.2$
$0.9 \leq r \leq 1.0$	$0.5R_0 - 787258.943t^{0.261} + 459546.025r^{2.834}$	$0.5R_0 - 694878.915t^{0.441} + 459546.025r^{2.834}$
$1.0 \leq r \leq 1.1$	$0.5R_0 - 787258.943t^{0.261} + 459546.025r^{0.578}$	$0.5R_0 - 694878.915t^{0.441} + 459546.025r^{0.578}$

Table 4. The equivalent resistance of the π-mode.

	$1.8 \leq t \leq 2.0$	$2.0 \leq t \leq 2.2$
$0.9 \leq r \leq 1.0$	$0.5R_0 + 787258.943t^{0.261} - 459546.025r^{2.834}$	$0.5R_0 + 694878.915t^{0.441} - 459546.025r^{2.834}$
$1.0 \leq r \leq 1.1$	$0.5R_0 + 787258.943t^{0.261} - 459546.025r^{0.578}$	$0.5R_0 + 694878.915t^{0.441} - 459546.025r^{0.578}$

the equivalent circuit parameters of the 0-mode coupled cavity, which can be expressed as the equivalent circuit parameters of the uncoupled cavities.

Ⅳ. Conclusion

We considered a simple equivalent parallel resonant circuit of the 1.5 cell RF gun resonated in the π-mode with the circuit parameters expressed as the functions depend on the thickness and the radius of the iris and the circuit parameters of the 0-mode, which can be expressed as the circuit parameters of the decoupled cavities again. The equivalent circuit model of the RF gun coupled cavity can be a useful tool in analyzing resonant characteristics of the coupled cavity in a steady state.

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