Dispersion and Attenuation Characteristics of Guided and Leaky Modes on Circular Lossy Dielectric Rod Waveguides

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ABSTRACT

Dispersion and attenuation characteristics of guided and leaky modes on circular lossy dielectric rod waveguides are analyzed for several lower order modes using an efficient and accurate Davidenko's complex root finding algorithm. In the leaky mode region, the normalized phase and attenuation constants of the lossless and lossy cases are very similar compared with the lossless case, if the dielectric is low loss, thereby the performance of the leaky wave antenna and other physical phenomena in the structure are expected to be little affected. The normalized phase constant of the guided mode has deflected to lower values compared with the lossless case as the operating frequency goes down, and finally the guided mode becomes cutoff or evolves into the next higher order leaky mode in the spectral gap region of that leaky mode. It is also observed that the attenuation constants obtained from the traditional perturbation method are meaningful only in some limited frequency ranges.

INTRODUCTION

Circular dielectric rod is one of the simplest guiding structures that can be used as either a low loss waveguide or a leaky wave antenna. Guided mode characteristics of the circular lossless dielectric rod waveguides have been well established for many decades and its leaky mode characteristics were also considered in [1]. Recently, more improved and detailed leaky dispersion characteristics of this structure have been analyzed in [2]. In order to get more general insight of this structure, the investigation on the dispersion characteristics of a lossy dielectric rod is strongly required. In this paper, the dispersion characteristics of the guided and leaky modes of the circular dielectric rod are investigated when the dielectric loss is introduced. We observe and discuss the effects of the dielectric loss on the dispersion and attenuation characteristics of the circular lossy dielectric rod such as a decrease in the physically meaningful frequency ranges of the leaky mode, a deflection of the normalized phase constant of the guided mode, a guided mode cutoff frequency shift phenomenon, a spectral broadening of the spectral gap region, and a mode coupling between the guided mode and the next higher order leaky mode. In addition, the attenuation constants obtained from the perturbation method are compared with those obtained from Davidenko's method.

CHARACTERISTIC EQUATION AND LEAKY WAVE

If we assume a time convention of $e^{j(\omega t-\gamma z)}$, the TM_{0n} mode characteristic equation of the circular lossy dielectric rod waveguide is given as follows.

$$\varepsilon_{r1c}k_2J_1(k_1a)H_0^{(2)}(k_2a)-\varepsilon_{r2}k_1J_0(k_1a)H_1^{(2)}(k_2a)=0 \qquad (1)$$

Here, $\varepsilon_{r1c}\left(=\varepsilon_{r1}(1-j\tan\delta)\right)$ is a complex dielectric constant of the dielectric rod and ε_{r2} is a real dielectric constant of the surrounding free space, *i.e.*, unity. k_1 and k_2 are the complex transverse propagation constant in the dielectric region and the free space region, respectively, and are related to the normalized complex axial propagation constant $\bar{\gamma}$ as $k_i^2=\left(\mathrm{Re}\{k_i\}+\mathrm{Im}\{k_i\}\right)^2=k_0^2(\mu_{ri}\varepsilon_{ri(c)}-\bar{\gamma}^2)$. The normalized complex axial propagation constants are defined as follows.

$$\bar{\gamma}=\frac{\gamma}{k_0}=\frac{\beta-j\alpha}{k_0}=\frac{\beta}{k_0}-j\frac{\alpha}{k_0}=\bar{\beta}-j\bar{\alpha} \qquad (2)$$

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The real and imaginary parts of the normalized complex propagation constants correspond to the normalized phase and attenuation constants, respectively, which are the phase and attenuation constants normalized by the free space wave number. Since the dielectric constant of the dielectric material is complex, the following relations should be satisfied.

$$\begin{cases} \left(\mathrm{Re}\{k_i\}\right)^2 - \left(\mathrm{Im}\{k_i\}\right)^2 = k_0^2\left(\mu_{ri}\varepsilon_{ri} - \overline{\beta}^2 + \overline{\alpha}^2\right) \\ 2\,\mathrm{Re}\{k_i\}\,\mathrm{Im}\{k_i\} = k_0^2\left(2\overline{\beta}\overline{\alpha} - \mu_{ri}\varepsilon_{ri}\tan\delta\right) \end{cases} \tag{3}$$

In addition, the condition $\overline{\beta} > 0$, $\overline{\alpha} > 0$, $\mathrm{Re}\{k_i\} > 0$, and $\mathrm{Im}\{k_i\} > 0$ should be satisfied for the complex waves to be the forward leaky waves. The characteristic equation (1) is numerically solved using Davidenko's method [3]. The obtained complex propagation constants are again substituted in the left hand side of Eq. (1) and the resultant values are compared with the zero. The absolute differences were kept under 10^{-10} for both the real and imaginary parts. In the analysis of this paper, the radius of the rod a, the real part of the complex dielectric constant ε_{r1} , and the loss tangent of the dielectric material $\tan\delta$ are assumed to be 10 mm, 10, and 0.01, respectively.

NUMERICAL RESULTS AND DISCUSSION

Figs. 1 (a) and (b) show the normalized phase and attenuation constants for several lower order guided mode and leaky modes. TM_{0n}^{G} and TM_{0n}^{L} in Fig. 1 stand for the guided TM_{0n} and leaky TM_{0n} modes, respectively. In the leaky mode region, the normalized phase constants of the lossy case are nearly the same but slightly higher than those of the lossless case, so that the physically meaningful frequency region ($\overline{\beta} < 1$) such as the antenna mode ($\overline{\beta} < 1$ and $\overline{\beta} > \overline{\alpha}$) and the reactive mode ($\overline{\beta} < 1$ and $\overline{\beta} < \overline{\alpha}$) regions [4] are slightly narrower than those of the lossless case. The normalized attenuation constants of the lossless and lossy cases in the leaky mode region are also very similar but there are several crossing points in the antenna mode region or in the spectral gap region. In the deep reactive region, there is an extraordinary phenomenon, *i.e.*, the normalized attenuation constants of the lossy case are little smaller than those of the lossless case in spite of the existence of the material loss. Unfortunately, the entire physical explanations about the crossing points between the normalized attenuation constants of the lossless and lossy cases and the smaller attenuations of the lossy case in the deep reactive region are not available at this time. Although there are differences of the normalized phase and attenuation constants between the lossless and lossy cases, the differences are so little if the loss of the practical dielectric material is low. Therefore, the performance of the circular dielectric rod leaky wave antenna such as a beam angle (depending on $\overline{\beta}$) and a beam width (depending on both $\overline{\beta}$ and $\overline{\alpha}$), and other physical phenomena in the leaky mode region are expected to be little affected compared with the lossless one. On the other hand, the propagation constants of the guided mode region become complex as well as those of the leaky mode region, since the dielectric constant is complex. In the Fig. 1 (a), as the operating frequency goes down, the normalized phase constants of the guided mode become lower compared with lossless case, and finally the guided modes are cutoff ($\overline{\alpha} = 0$) or are transited into the next higher order leaky modes. The cutoff frequencies of the guided modes are shifted toward certain higher frequencies at which the normalized phase constants of the next higher leaky modes have the maximum values in the spectral gap region ($\overline{\beta} > 1$). The shifted cutoff frequencies of the guided modes are shown in Fig. 1(b). Figs. 2 (a) and (b) show the magnified plots of the normalized phase and attenuation constants near to the spectral gap of the TM_{03}^{L} mode. The spectral width of the spectral gap becomes broader than that of the lossless case. The deflected normalized phase constant of the TM_{02}^{G} mode approaches the normalized phase constant of the TM_{03}^{L} mode, but TM_{02}^{G} mode is cutoff before contacting to the TM_{03}^{L} mode. In the frequency region near to the spectral gap of the TM_{02}^{L} mode, the similar situation occurs. Figs. 3 (a) and (b) show the magnified plots of the normalized phase and attenuation constants near to the spectral gap of the TM_{04}^{L} mode. The TM_{03}^{G} mode continuously evolves into the TM_{04}^{L} mode near to the peak point in the spectral gap of TM_{04}^{L} mode. In the encircled region in the Figs. 3 (a) and (b), the mode coupling between the TM_{03}^{G} mode and the TM_{04}^{L} mode occurs. Since the mode coupling between the guided mode and the next higher order leaky mode evolves continuously, the critical frequency of the transition is difficult to determine. The normalized phase and attenuation constants of the TM_{02}^{G} (or TM_{01}^{G}) and TM_{03}^{L} (or TM_{02}^{L}) modes have different value, so that any mode coupling does not occur in the spectral gap of the TM_{03}^{L} (or TM_{02}^{L}) mode. At the peak value of the normalized phase constants, two real and nonspectral solutions are nonphysical in lossless case [5]. In our lossy case, however, the larger one from the two solutions becomes complex but physical, and it represents the next lower order guided mode solution. We also compared the normalized attenuation constants obtained from Davidenko's method and from the traditional perturbation method [6]. The results are shown in Fig. 5. Note that the perturbation method can only be applied in some limited frequency ranges, such as, for example, the frequency ranges of the encircled regions in Fig. 5.

CONCLUSIONS

We have analyzed the dispersion and attenuation characteristics of the guided and leaky dispersion characteristics on the circular lossy dielectric rod waveguides by using efficient and accurate Davidenko's method. Several interesting features influenced by the dielectric loss such as the decrease of the physically meaningful leaky mode region, the increase of the spectral width of the spectral gap, the guided mode cutoff frequency shift, and the mode coupling between the guided mode and the next higher order leaky mode in the spectral gap of that leaky mode are observed and discussed. We also compared the attenuation constants obtained from the perturbation method and Davidenko's method.

Acknowledgement

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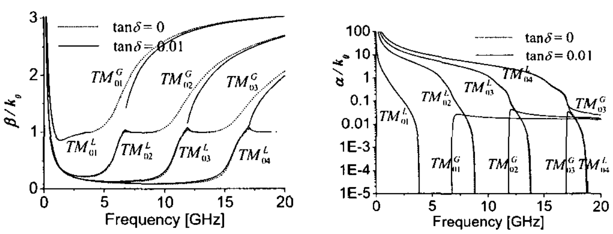

(a) (b)

Fig. 1. (a) Normalized phase constant and (b) Normalized attenuation constants of the circular dielectric rod employed in this work.

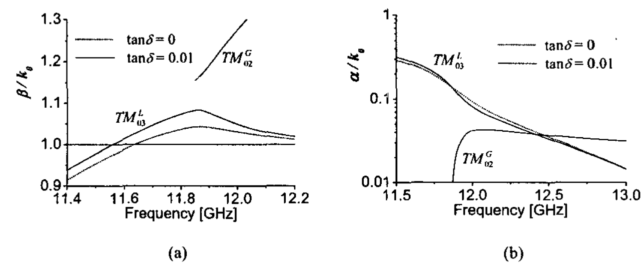

Fig. 2. Expanded plots of (a) the normalized phase constants and (b) the normalized attenuation constants near at the spectral gap of TM_{03}^{L} mode.

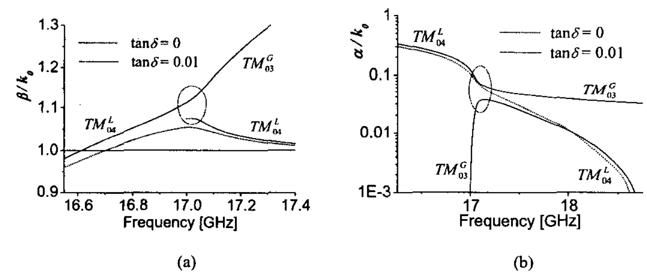

Fig. 3. Expanded plots of (a) the normalized phase constants and (b) the normalized attenuation constants near at spectral gap of TM_{04}^{L} mode. The mode coupling between the TM_{03}^{G} mode and the TM_{04}^{L} mode occurs in the encircled region.

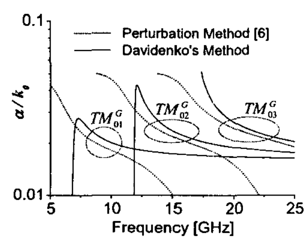

Fig. 4. Normalized attenuation constants obtained from the perturbation method and Davidenko's method.