

# DGS Dual Composite Right/Left Handed Transmission Line

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**Abstract**—The planar dual composite right/left-handed transmission line (D-CRLH TL) using a defected ground structure (DGS) and stub with rectangular patch is presented. The required series parallel resonance tank and shunt series resonance tank for the D-CRLH TL are provided by DGS and stub with rectangular patch, respectively. Its equivalent circuit is provided to analyze D-CRLH TL. The characteristics such as dispersion relation and frequency response of D-CRLH TL are analyzed by circuit analysis, Bloch-Floquet theory, and full wave simulation. The measured results of dispersion curves and frequency behaviors have good agreements with theory. Finally, it is shown that compact multiband resonator using proposed D-CRLH TL can be realized.

**Index Terms**—Defected ground structure (DGS), dual composite right/left-handed transmission line (D-CRLH TL), stub with rectangular patch.

## I. INTRODUCTION

OVER the past decade, in the electromagnetic communities, metamaterials have evolved with novel electromagnetic concepts such as backward wave propagation, negative index refraction, and infinite wavelength wave propagation [1]. From a practical application point of view, planar meta-structured transmission lines [2] are broadly applied to various RF devices due to low loss, broad bandwidth, convenience of theoretical analysis, and simplicity of the fabrication. Recently, the novel concepts of dual composite left/right handed (D-CRLH) and higher-order metamaterial transmission lines are introduced for multi-band application [3], [4]. In particular, the D-CRLH consists of a series parallel  $LC$  resonance tank and a shunt series  $LC$  resonance tank having duality concept of CRLH TL. The D-CRLH TL is more advantageous to multi-band application than CRLH TL because its dispersion curve has three branches in half periodicity. Therefore, it can simply be applied to the multi band radio frequency (RF) devices.

In this letter, the planar D-CRLH TL is presented using a defected ground structure (DGS) [5], [6] and stub with rectangular patch. DGS and stub with rectangular patch are very good

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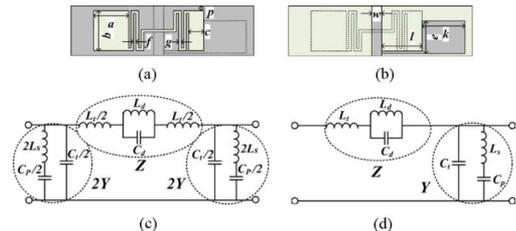


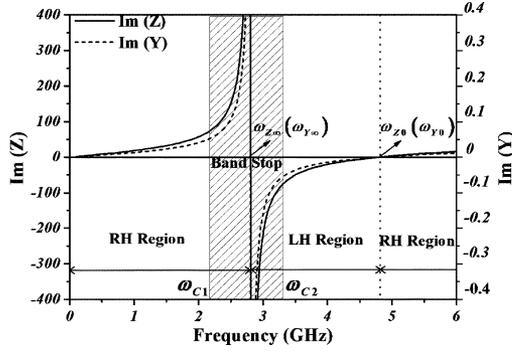
Fig. 1. Unit cell of proposed DRLH-TL and its equivalent circuits ( $a = 3$  mm,  $b = 3$  mm,  $c = 1$  mm,  $e = 2.2$  mm,  $f = 0.2$  mm,  $g = 0.2$  mm,  $k = 3$  mm,  $l = 2.4$  mm,  $w = 0.6$  mm, and  $p = 0.5$  mm) (a) DGS on ground (bottom). (b) Stub with rectangular patch (top). (c) Equivalent  $\pi$  network. (d) Equivalent ladder network.

unit-cell elements to provide the required series parallel  $LC$  resonance tank and shunt series  $LC$  resonance tank of D-CRLH because they are perfectly planar without via hole and can be easily fabricated by standard etching technique. In addition, the desired RH-LH-RH regions of D-CRLH can be controlled by adjusting the dimensions of unit-cell elements. It will be shown that compact multiband resonator can be constructed with proposed D-CRLH TL.

## II. STRUCTURE AND EQUIVALENT CIRCUIT OF PLANAR D-CRLH TL

Fig. 1 shows the proposed D-CRLH TL structure and equivalent circuit of the unit cell. The unit cell consists of DGS on ground and stub with rectangular patch on signal line. DGS is realized by etching a defected pattern on the ground plane of 1-D transmission line as shown in Fig. 1(a). The pattern usually adds an extra lumped capacitance and inductance to the host transmission lines [5], [6]. Then, DGS is equivalently modeled as a parallel resonant circuit connected in series on host transmission line as shown in Fig. 1(c). The DGS consists of interdigital gap and etched rectangular patch to lower the resonant frequency by increasing the capacitance value. Stub with rectangular patch on signal line is shown in Fig. 1(b). Since the electric length of the stub is less than  $\lambda/2$  for the interested frequency range, it can be represented by an inductance. Since the stub is terminated with a rectangular patch, the stub with rectangular patch is modeled as a shunt series resonance tank as shown in Fig. 1(c). Naturally, these two structures have band-stop characteristic with negative permeability and permittivity, respectively. Thus, D-CRLH TL can be constructed with a combination of two structures.

In Fig. 1(c), the capacitance ( $C_d$ ) and inductance ( $L_d$ ) of the series parallel resonance tank depend on the number of the interdigital finger and/or their gap width and the etched rectangular size, respectively. Also, the capacitance ( $C_p$ ) and inductance ( $L_s$ ) of the shunt series resonance tank depend on the dimension


 Fig. 2. Imaginary values of  $Z$  and  $Y$ .

of rectangular patch and stub, respectively. The capacitance ( $C_t$ ) and inductance ( $L_t$ ) are parasitic elements of D-CRLH since the host transmission line has an intrinsic series inductance ( $L_t$ ) and shunt capacitance ( $C_t$ ). Consequently,  $\pi$ -equivalent circuit is completed as shown in Fig. 1(c). To easily obtain dispersion relation of the periodic structure, the  $\pi$ -network is changed to ladder network as shown in Fig. 1(d). In the model, each parameter is extracted from the circuit and full wave simulation. When D-CRLH TL is designed with the balanced condition which will be described in Section III, the extracted circuit parameters of the unit cell are  $L_d = 1.79$  nH,  $C_d = 1.81$  pF,  $L_s = 2.51$  nH,  $C_p = 1.29$  pF,  $L_t = 0.93$  nH, and  $C_t = 0.665$  pF, respectively. These values of inductances and capacitances can be controlled by adjusting aforementioned unit cell parameters.

### III. THEORY

The fundamental characteristics of D-CRLH TL are straightforwardly analyzed by elementary TL theory and Bloch–Floquet theorem. In Fig. 1(d), the series impedance ( $Z$ ), shunt admittance ( $Y$ ), and characteristic impedance ( $Z_C$ ) are derived [3]

$$Z(\omega) = j\omega L_t \frac{\omega^2 - \omega_{Z0}^2}{\omega^2 - \omega_{Z\infty}^2}, Y(\omega) = j\omega C_t \frac{\omega^2 - \omega_{Y0}^2}{\omega^2 - \omega_{Y\infty}^2} \quad (1a)$$

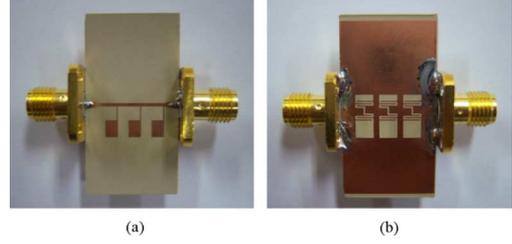
$$Z_C(\omega) = \sqrt{\frac{Z(\omega)}{Y(\omega)}} = \sqrt{\frac{L_t}{C_t}} \sqrt{\frac{(\omega^2 - \omega_{Z0}^2)(\omega^2 - \omega_{Y\infty}^2)}{(\omega^2 - \omega_{Z\infty}^2)(\omega^2 - \omega_{Y0}^2)}} \quad (1b)$$

where

$$\omega_{Z\infty} = \frac{1}{\sqrt{C_d L_d}}, \quad \omega_{Y\infty} = \frac{1}{\sqrt{C_p L_s}}$$

$$\omega_{Z0} = \sqrt{\frac{L_t + L_d}{C_d L_d L_t}}, \quad \omega_{Y0} = \sqrt{\frac{C_t + C_p}{C_p L_s C_t}}.$$

Then, the effective permeability and permittivity values are obtained from  $\mu_{eff} = Z(\omega)/j\omega$  and  $\epsilon_{eff} = Y(\omega)/j\omega$ . Fig. 2 shows the imaginary values of the per-unit length impedance and admittance with the balanced condition versus frequency.  $\text{Im}(Z)$  values can be divided into three bounded regions by the critical frequency points of  $\omega_{Z\infty}$  and  $\omega_{Z0}$ . Similarly,  $\text{Im}(Y)$  values have three bounded regions divided by  $\omega_{Y\infty}$  and  $\omega_{Y0}$ . In Fig. 2, below  $\omega_{Z\infty}(\omega_{Y\infty})$ , the series impedance and shunt admittance have positive values and, thus, D-CRLH TL has RH band. Between  $\omega_{Z\infty}(\omega_{Y\infty})$  and  $\omega_{Z0}(\omega_{Y0})$ , the series impedance (shunt admittance) has negative value. This indicates that the region is effectively double negative. Thus, it


 Fig. 3. Fabricated three-stage D-CRLH TL. (a) Top view. (b) Bottom view (Rogers RO3010 substrate:  $h = 0.64$  mm,  $\epsilon_r = 10.2$ ).

has LH characteristic because  $C_t$ ,  $C_d$ ,  $L_s$ , and  $C_p$  are dominant elements. Finally, above  $\omega_{Z0}(\omega_{Y0})$ , it has the second RH characteristic since the series impedance and shunt admittance have positive values. D-CRLH TL has two balanced conditions for broadband matching. The conditions are

$$\omega_{Z\infty} = \omega_{Y\infty} \Rightarrow 2.78 \text{ GHz and } \omega_{Z0} = \omega_{Y0} \Rightarrow 4.8 \text{ GHz.} \quad (2)$$

In this case, using extracted circuit parameters, the characteristic impedance is obtained by (1)

$$Z_C(\omega) = \sqrt{\frac{Z(\omega)}{Y(\omega)}} \approx \sqrt{\frac{L_t}{C_t}} \approx \sqrt{\frac{L_s}{C_d}} \approx \sqrt{\frac{L_d}{C_p}}. \quad (3)$$

Therefore, the characteristic impedance of D-CRLH TL is to be purely real value in the whole frequency range and broadband impedance matching can be achieved. By applying the periodic boundary conditions related with Bloch–Floquet theorem to the unit cell, the dispersion relation is obtained as

$$\beta(\omega) = \frac{1}{d} \cos^{-1} \left( 1 + \frac{ZY}{2} \right) \quad (4)$$

where  $\beta$  is propagation constant for Bloch waves and  $d$  is the periodicity of the structure. Near  $\omega_{Z\infty}(\omega_{Y\infty})$ , values of  $\text{Im}(Z)$  and  $\text{Im}(Y)$  are much larger than one, so that the propagation constant in (4) has imaginary value. Thus, D-CRLH has stop band as shown in Fig. 2.  $\omega_{c1}$  and  $\omega_{c2}$  are cutoff frequency in RH band and in LH band, respectively. The cutoff frequencies of  $\omega_{c1}$  and  $\omega_{c2}$  are calculated using (1) and the condition  $\beta d = 0$  (or  $\pi$ ) of (4). The resulting equation is

$$L_t C_t \omega^6 - (L_t C_t \omega_{Z0}^2 + L_t C_t \omega_{Y0}^2 + 4) \omega^4 + (L_t C_t \omega_{Z0}^2 \omega_{Y0}^2 + 4\omega_{Z\infty}^2 + 4\omega_{Y\infty}^2) \omega^2 - 4\omega_{Z\infty}^2 \omega_{Y\infty}^2 = 0. \quad (5)$$

Equation (5) is a cubic equation with respect to  $\omega^2$  and three roots of this equation can be obtained by Cardano's formula. The obtained cutoff frequencies ( $\omega_{c1}$ ,  $\omega_{c2}$ ,  $\omega_{c3}$ ) are 2.174 GHz, 3.305 GHz, and 13.93 GHz. Note that  $\omega_{c3}$  is the cutoff frequency in the second RH band.

### IV. SIMULATION AND EXPERIMENT

Fig. 3 shows the top and bottom views of the fabricated three-stage D-CRLH TL. The input and output ports are fabricated using 50  $\Omega$  coaxial connector. The width of the signal line is 0.6 mm corresponding to a 50  $\Omega$  line while the characteristic impedance of D-CRLH TL is 37.3  $\Omega$  from (3). So, the width and length of feed line are optimized as 1.05 mm and 3 mm for impedance matching, respectively.

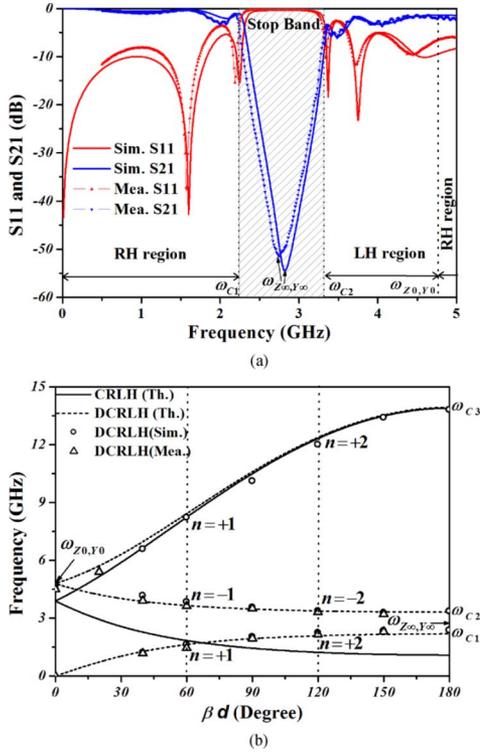


Fig. 4. (a) Frequency behavior for three stages D-CRLH TL. (b) Dispersion curves of CRLH and D-CRLH TL.

Fig. 4 shows the simulated and measured S-parameters for three-stage D-CRLH TL and dispersion curves by theorem, simulation, and measurement. As shown in Fig. 4(a), RH, stop, LH and second RH band are measured. The measurements results are in very good agreement with those of simulation. These results are also confirmed in the dispersion diagram as shown in Fig. 4(b). The obtained dispersion curves from theory using (1) and (4), full wave simulation of assuming the infinite periodic structure, and measurement have been compared in Fig. 4(b). To compare the dispersion characteristics of CRLH TL and D-CRLH TL, the dispersion curve of CRLH TL is over plotted in Fig. 4(b), assuming that  $L_d$  and  $C_p$  are zero in Fig. 1(d) to be a CRLH TL. As expected, the D-CRLH TL has an advantage in multi-band application compared with CRLH TL because its dispersion curve has three branches while CRLH TL has two branches in half periodicity. In spite of the existence of the defected structure on the ground plane and stub with rectangular patch on signal line, the radiation loss could be ignored since it was measured to be less than  $-17.5$  dB in the whole frequency band.

To show that the proposed D-CRLH TL can be utilized for compact multiband resonator, the resonances of D-CRLH TL for resonance modes  $n$  can also be obtained by

$$\beta_n d = \frac{n\pi d}{l} = \frac{n\pi}{N} \quad (6)$$

$$\begin{cases} n = +1, +2, \dots, +(N-1) & \text{in RH band} \\ n = -1, -2, \dots, -(N-1) & \text{in LH band} \\ n = 0 & \text{at the boundary} \\ n = +1, +2, \dots, +(N-1) & \text{in second RH band} \end{cases}$$

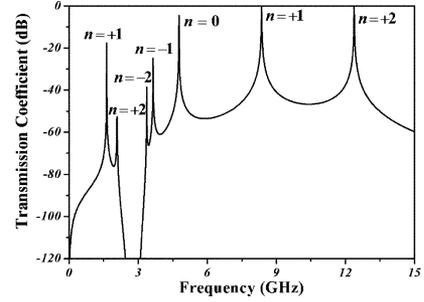


Fig. 5. Simulated resonance modes of three-stage D-CRLH resonators.

where  $N (= l/d)$  and  $l$  are the number of unit cell and total length of resonator, respectively. Thus, it is straightforward that D-CRLH TL has many modes in comparison with CRLH TL. The three-stage resonator using open-ended boundary ( $N = 3$ ) is simulated by circuit simulator. Fig. 5 shows the resonance modes of the resonator. As stated in (6), three-stage D-CRLH resonator generates two negative, four positive, and one zeroth resonance modes, which are all less than half wavelength. In Fig. 5, the simulated (theoretical) resonance mode frequencies of the resonator are obtained as 1.64 (1.63) GHz, 2.08 (2.09) GHz, 3.37 (3.38) GHz, 3.65 (3.66) GHz, 4.77 (4.80) GHz, 8.37 (8.42) GHz, and 12.38 (12.37) GHz, respectively. The results indicates that our proposed D-CRLH TL can be simply applied to the multiband RF devices such as antennas, resonators, and filters.

## V. CONCLUSION

Using DGS and stub with rectangular patch of the perfectly planar type, the DGS D-CRLH TL is proposed. The characteristics of proposed TL are analyzed by dispersion relation and frequency behavior using theory, simulation, and experimentation. The measured results of the D-CRLH TL have good agreements with those of simulation and theory. The radiation loss is simulated as about  $-17.5$  dB in maximum case. The radiation loss is small enough to be ignored since it was measured to be less than  $-17.5$  dB in the whole frequency band. Finally, it has been shown that compact multiband resonator can be constructed with proposed D-CRLH TL.

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